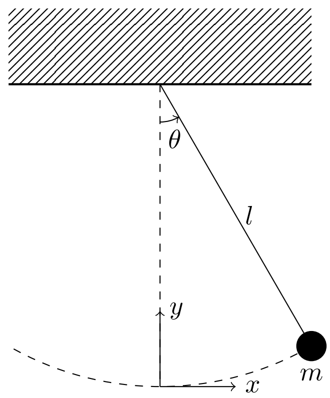
PHYS 2601: Classical and Quantum Waves Midterm

Professor James McIver

Problem 1 (20 points)

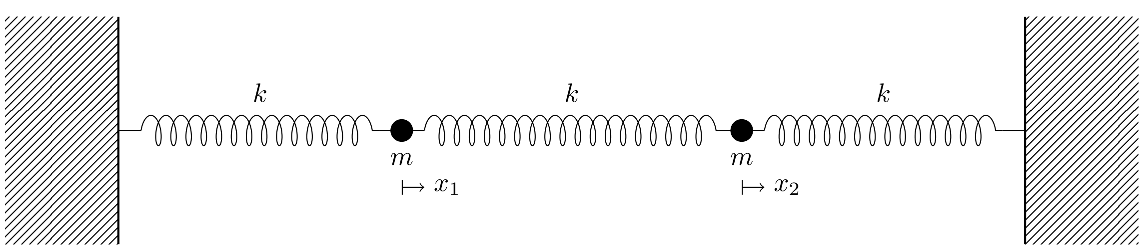


Suppose we have a simple pendulum of mass m attached to a massless rod of length l. The angular dependence of the position can be described as follows:

$$x(\theta) = l \sin \theta$$
$$y(\theta) = l - l \cos(\theta)$$

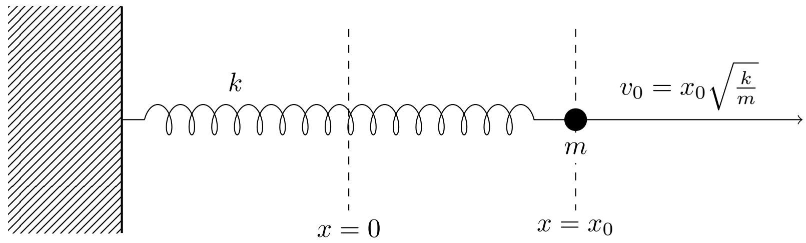
a) Use the Taylor approximation of $\sin \theta \approx \theta$ and $\cos \theta \approx 1 - \theta^2/2$ to find the height of the mass in terms of its x position for small values of theta (find $y(x)$). (5 points)
b) Using $y(x)$ and the gravitation potential energy function $U(h) = mgh$, where $h = y(x)$, to find the force on the mass for small theta. (5 points)
c) Write the differential equation of motion for this system using the force from part **b**. (5 points)
d) What is the angular frequency ω of the motion of this system? (5 points)

Problem 2 (30 points)



Suppose we have a coupled oscillator system as modeled above. Each spring has an identical spring constant k and the masses are also identical.
a) Write the forces on each mass (make sure the signs are correct). (5 points)
b) Using the results from part **a**, write the differential equation of motion for each mass. Explain why these two equations are coupled. (5 points)
c) Rewrite the two equations of motion from part **b** in terms of $q_1 = x_1 + x_2$ and $q_2 = x_1 - x_2$. (5 points)
d) Using the uncoupled equations of motion of q_1 and q_2 **or** the matrix form of x_1 and x_2, solve for the angular frequencies associated with the normal modes of the system. (10 points)
e) Describe the motion of the masses as they oscillate at each normal frequency. (5 points)

Problem 3 (25 points)



A mass m attached to a spring with constant k oscillates about $x = 0$ as shown in the diagram above. The mass is also subject to a a **very light** damping force $F_d = -bv = -b\dot{x}$.
a) Write the differential equation modeling the motion of the mass. (5 points)
b) Assume that the system is lightly damped. At time $t = 0$ the mass is at position $x = x_0$ and moving with velocity $\dot{x} = x_0\sqrt{\frac{k}{m}}$. Using the Ansatz for damped simple harmonic motion, solve the equation of motion using the initial conditions. What is the approximate amplitude A_0? You don't need to solve for a phase offset ϕ. Make sure to define any new quantities you introduce in your solution. (10 points)
c) How does the phase compare to the case where $v = 0$ at $t = 0$? Give the sign and approximate value. (5 points)
d) Derive the initial energy E_0 of the system in terms of A_0. (HINT: Directly use the results from part **b**) (5 points)

Problem 4 (15 points)

a) For a driven oscillator show that the energy dissipated per cycle by a frictional force $F_d = -bv$ at frequency ω and amplitude A is equal to $\pi b\omega A^2$. (5 points)
b) Hence show (5 points)

$$\frac{\text{energy dissipated/cycle}}{\text{stored energy}} = \frac{2\pi b}{m\omega}.$$

c) Show that at the resonance frequency of a lightly damped oscillator (5 points)

$$\frac{\text{energy dissipated/cycle}}{\text{stored energy}} = \frac{2\pi}{Q}.$$